Homework #6, PHY 674, 16 October 1995

nsider the following hypothetical molecule: It consists of three hydrogen atoms located at the corners an equilateral triangle. Find all irreducible characters of the corresponding (3-dimensional) symmetry up and their dimensions. Briefly describe how you found the characters. Which degeneracies are owed for the electronic states of this system ? Assuming that $\psi_{1s}(i)$ is a $1s$-wave function centered the i-th corner, project out the components of $\psi_{1s}(1)$ belonging to the irreducible representations. hich of these wave functions do you believe to have the lowest energy, and why ? (4 points).

ıe benzene molecule C_6H_6 is a hexagonal ring of 6 carbons atoms, each of which has a hydrogen atom :ached. All atoms are located in the same plane. Find the (3-d) symmetry group for this molecule d determine the character table. Describe how you can find the irreducible characters (4 points).

ove the Clebsch-Gordan formula for the group SU (2) (4 points):

$$V_k \otimes V_l \;=\; \bigoplus_{j=0}^{q} V_{k+l-2j} \quad \text{with} \quad q = \min\{k,l\}\,. \tag{27.1}$$

ove the Clebsch-Gordan formula for the group SO (3) (4 points):

$$W_k \otimes W_l \;=\; W_{|k-l|} \oplus W_{|k-l|+1} \ldots \oplus W_{k+l}. \tag{28.2}$$

ow that the group O (3) of orthogonal symmetry operations in three dimensions is a direct product (3) = SO (3) $\times \{E, \pi\}$, where E is the identity and π the inversion. Use this knowledge to find the ısses and irreducible characters of O (3) (4 points).

compose the representation denoted by Γ in the table given below into the irreducible representations the group labelled T_d.

T_d	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$
Γ_1	1	1	1	1	1
Γ_2	1	1	1	−1	−1
Γ_3	2	−1	2	0	0
Γ_4	3	0	−1	1	−1
Γ_5	3	0	−1	−1	1
Γ	9	0	1	−1	3

ease note that this notation for the irreducible representations of T_d is somewhat unusual, at least semiconductor physics (4 points).